Open inflationary universes in a brane world cosmology

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In this paper, we study a type of one-field model for open inflationary universe models in the context of brane world models. In the scenario of a one-bubble universe model, we determine and characterize the existence of the Coleman–De Lucia instanton, together with the period of inflation after tunneling has occurred. Our results are compared to those found in the Einstein theory of relativistic models.

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I. INTRODUCTION

Recent observations from the WMAP [1] are entirely consistent with a universe having a total energy density that is very close to its critical value, where the total density parameter has the value $\Omega = 1.02 \pm 0.04$. Most people interpret this value as corresponding to a flat universe. But according to this result, we might take the alternative point of view of having a marginally open or closed universe model [2], which early presents an inflationary period of expansion. This approach has already been considered in the literature ([3,4] or [5,6]) in the context of the Einstein theory of relativity or a Jordan-Brans-Dicke (JBD) type of gravitation theory, respectively. All these models have been worked out in a four-dimensional spacetime. At this point, we should mention that Ratra and Peebles were the first to elaborate on the open inflation model [7].

The idea of considering an extra dimension has received great attention in the last few years, since it is believed that these models shed light on the solution to fundamental problems when the universe is traced back to a very early time. Specifically, the possibility of creating an open or closed universe from the context of a brane world (BW) scenario [8] has quite recently been considered.

BW cosmology offered a novel approach to our understanding of the evolution of the universe. The most spectacular consequence of this scenario is the modification of the Friedmann equation, in a particular case when a five-dimensional model is considered and where the matter described through a scalar field is confined to a four-dimensional brane, while gravity can be propagated in the bulk. These kinds of models can be obtained from a higher superstring theory [9]. For a comprehensible review of BW cosmology, see [10], for example. Specifically, the consequences of a chaotic inflationary universe scenario in a BW model was described [11], where it was found that the slow-roll approximation is enhanced by the modification of the Friedmann equation. The purpose of the present paper is to study an open inflation universe model, where the scalar field is confined to the four-dimensional brane.

The plan of the paper is as follows: In Sec. II we specify the effective four-dimensional cosmological equations from a five-AdS BW model. We write down the field equations in a Euclidean spacetime, and we solve them numerically. Here, the existence of the Coleman–De Lucia (CDL) instanton for two different models is described. In Sec. III we determine the characteristic of an open inflationary universe model that is produced after tunneling has occurred. In Sec. IV we determine the corresponding density perturbations for our models. In any case, our results are compared to those analogous results obtained by using Einstein's theory of gravity. Finally, we conclude in Sec. V.

II. EUCLIDEAN COSMOLOGICAL EQUATIONS IN RANDALL-SUNDRUM TYPE-II SCENARIO

We start with the action given by

$$S = M_5^3 \int d^5x \sqrt{-G}\,({}^{(5)}R - 2\Lambda_5) - \int d^4x \sqrt{-g}\,\mathcal{L}_{matter}, \tag{1}$$

where

$$\mathcal{L}_{matter}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)$$

describes the matter confined in the brane, ${}^{(5)}R$ is the Ricci scalar curvature of the metric G_{ab} of the five-dimensional bulk, and M_5 and Λ_5 denote the five-dimensional Planck mass and cosmological constant, respectively. The following relations are found to be valid in this case:

$$\Lambda_4 = \frac{4\pi}{M_5^3}\left(\Lambda_5 + \frac{4\pi}{3M_5^3}\sigma^2\right) \tag{2}$$

and

$$M_4 = \sqrt{\frac{3}{4\pi}}\left(\frac{M_5^2}{\sqrt{\sigma}}\right) M_5, \tag{3}$$

where Λ_4 represents the effective cosmological constant on to brane and σ corresponds to the brane tension.

For this theory, Shiromizu *et al.* [12] have shown that the four-dimensional Einstein equations induced on the brane can be written as

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left(\frac{8\pi}{M_4^2}\right) T_{\mu\nu} + \left(\frac{8\pi}{M_5^2}\right) S_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{4}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the matter in the brane, $S_{\mu\nu}$ is the local correction to standard Einstein equations due to the extrinsic curvature, and $\mathcal{E}_{\mu\nu}$ is the nonlocal effect corrections from a free gravitational field, which arises from the projection of the bulk Weyl tensor. These quantities are given by

$$S_{\mu\nu}=\frac{1}{12}\rho^2 u_\mu u_\nu+\frac{1}{12}(\rho+2p)(g_{\mu\nu}+u_\mu u_\nu), \tag{5}$$

where ρ and p represent the energy density and pressure of a fluid, respectively, and

$$\mathcal{E}_{\mu\nu}=\left(\frac{M_4^2}{8\pi}\right)^2\left(\frac{A}{a^4}u_\mu u_\nu+\pi_{\mu\nu}\right), \tag{6}$$

where A is a constant and $\pi_{\mu\nu}$ is the anisotropic stress. Since we are considering an AdS_5 bulk and a Friedman-Robertson-Walker (FRW) brane, we should have $\pi_{\mu\nu}=0$. On the other hand, an extended version of Birkhoff's theorem tells us that if the bulk spacetime is AdS, then $\mathcal{E}_{\mu\nu}=0$ [10,13]. Finally, we can put $\Lambda_4=0$, when a fine-tuning is made over Λ_5.

In order to write down the field equations, recall that, due to Bianchi identity—i.e., $\nabla^\mu G_{\mu\nu}=0$—we have

$$\left(\frac{8\pi}{M_4^2}\right)\nabla^\mu T_{\mu\nu}+\left(\frac{8\pi}{M_5^2}\right)\nabla^\mu S_{\mu\nu}-\nabla^\mu\mathcal{E}_{\mu\nu}=0. \tag{7}$$

The first term is automatically satisfied and thus the following constraint is thus obtained:

$$\nabla^\mu\mathcal{E}_{\mu\nu}=\left(\frac{8\pi}{M_5^2}\right)\nabla^\mu S_{\mu\nu}. \tag{8}$$

Now, since in our case we have $\mathcal{E}_{\mu\nu}=0$, Eq. (8) becomes [11]

$$\nabla^\mu S_{\mu\nu}=0. \tag{9}$$

The $O(4)$ invariant Euclidean spacetime metric is written as

$$ds^2=d\tau^2+a(\tau)^2(d\psi^2+\sin^2\psi d\Omega_2^2). \tag{10}$$

The scalar field equation becomes

$$\phi''+3\frac{a'}{a}\phi'-V_{,\phi}(\phi)=0, \tag{11}$$

where $a(\tau)$ is the scale factor, the prime represents a derivative with respect to the Euclidean time (τ), and $V_{,\phi}(\phi)=dV/d\phi$.

When the metric (10) is introduced into Eq. (4), we obtain the following field equations for scalar factor:

$$\left(\frac{a'}{a}\right)^2=\frac{1}{a^2}-\frac{8\pi}{3M_4^2}\rho_E\left(1+\frac{\rho_E}{2\sigma}\right), \tag{12}$$

where ρ_E corresponds to Euclidean energy density associated with the scalar field, $\rho_E=-\frac{1}{2}\phi'^2+V(\phi)$. From now on we will use units where $M_4=G_4^{-1/2}=1$ and $\kappa=8\pi/M_4^2=8\pi$.

From Eqs. (11) and (12) we obtain

$$\frac{a''}{a}=-\frac{8\pi}{3M_4^2}\left(\phi'^2+V(\phi)+\frac{1}{8\sigma}[5\phi'^2+2V(\phi)][-\phi'^2+2V(\phi)]\right). \tag{13}$$

We consider the effective scalar potential $V(\phi)$ to be of a form analogous to that described in Ref. [6]:

$$V(\phi)=\frac{\lambda_n\phi^n}{2}\left(1+\frac{\alpha^2\tanh(v_n-\phi)}{\beta^2+(\phi-v_n)^2}\right), \tag{14}$$

where α, β, and v_n are arbitrary constants. We will take the particular values $n=2$ and $n=4$ from this potential. The second term controls the bubble nucleation. Its role is to create an appropriate shape in the inflaton potential $V(\phi)$ with a maximum value near $\phi=v_n$. The first term controls inflation after quantum tunneling has occurred, and its shape coincides with that used in the simplest chaotic inflationary universe model, $m^2\phi^2/2$. Following Ref. [6] we take $\alpha^2=0.1$ and $\beta^2=0.01$. Certainly, this is not the only choice, since other values for these parameters can also lead to a successful open inflationary scenario (with any value of Ω, in the range $0<\Omega<1$).

We have numerically solved the field equations (11) and (13) for the values $n=2$ and $n=4$ in the effective potential (14). However, the instanton has the topology of a four-sphere, and there are two places at which $a=0$. These are the points at which $\tau=0$ and $\tau=\tau_{max}$. Then, the boundary conditions on ϕ arise from the requirement that $3\dot{\phi}\dot{a}/a$ be finite—i.e., $\dot{\phi}(0)=\dot{\phi}(\tau_{max})=0$. From Eq. (12), we obtain that, at the zeros of the scalar factor, $\dot{a}=\pm1$. Since we have used units where the Planck mass in four dimensions is equal to 1, then the Planck mass in five dimensions becomes $M_5\leqslant10^{-2}$ [11] and by means of Eq. (3) we arrive at $\sigma=10^{-10}$.

On the other hand, we take the value of the constant λ_n in such a way that an appropriated amplitude for density perturbation is obtained. Thus, we take the values $\lambda_2=1.5\times10^{-6}$ and $\lambda_4=10^{-14}$. We choose $v_2=3.5$ and $v_4=4.8$, since they provide the needed 60 e-folds of inflation after tunneling has occurred.

At $\tau\approx0$, the scalar field $\phi=\phi_T$ lies in the true vacuum, near the maximum of the potential, which (in Euclidean signature) corresponds to $-V(\phi)$. At $\tau\neq0$, the same field is found close to the false vacuum, but now with a different value, $\phi=\phi_F$. Specifically, for $n=2$ and $n=4$ and when the scalar field ϕ evolves from some initial value—i.e., $\phi_F\cong\phi_i\approx3.52$ to the final value $\phi_T\cong\phi_f$—numerically we have found that the CDL instanton does exist and the brane world open inflationary universe scenario can be realized. Table I summarizes our results, which are compared with those corresponding to Einstein's theory of relativity.

Note that the interval of tunneling, specified by τ, decreases when the $n=4$, but its shapes remain practically

TABLE I. This table shows the values of ϕ_F and ϕ_T for which the CDL instanton exists.

Models	ϕ_F	ϕ_T
$n=2$ GR[a]	3.52	3.31
$n=2$ BW	3.52	3.32
$n=4$ GR	4.85	4.60
$n=4$ BW	4.88	4.60

[a]GR = general relativity.

similar. The evolution of the inflaton field as a function of the Euclidean time is shown in Fig. 1.

In Fig. 2 we show $|V''|/H^2$ as a function of the Euclidean time τ for our model. From this plot we observe that, most of the time during the tunneling, we obtain $|V''|>H^2$, analogous to what occurs in Einstein's GR theory. Note that, for $n=4$, the peak becomes narrower and deeper, and thus the above inequality is better satisfied.

It is numerically possible to show that the CDL instanton $\phi(\tau)$ exists for various values of the σ parameter. The principal difference shown is the deviation to general relativity, but this solution presents a similar behavior to that described by Linde [3]. The values coincide for large τ, and its values (after tunneling has occurred) coincide in the two theories—i.e., Einstein's GR and BW theories.

The following expression represents the instanton action for the quantum tunneling between the false and true vacuums in the BW theory. In order to reproduce the field equations of motions (11)–(13) and using the constraint (9), we write, for the instanton action,

$$S=2\pi^2\int d\tau\left[a^3\left(\frac{1}{2}\phi'^2+V(\phi)\right)+\frac{a^3}{2\sigma}\left(\frac{1}{2}\phi'^2+V(\phi)\right)^2+\frac{3}{\kappa}(a^2a''+aa'^2-a)\right]. \tag{15}$$

We should note that this action is not obtained from Eq. (1). At the moment, as far we know nobody has performed this task. Integrating by parts and using the Euclidean equations of motion, we find that the action may be written as

$$S=4\pi^2\int d\tau\left[a^3V\left(1+\frac{V}{2\sigma}\right)+\frac{a^3}{8\sigma}\phi'^4-\frac{3a}{\kappa}\right]. \tag{16}$$

Note that this action coincides with that corresponding to its analogue in Einstein's general relativity theory if we take the limit that $\sigma\to\infty$ [14].

The inflaton field ϕ is initially trapped in its false vacuum, and a value specified by ϕ_F is obtained. After tunneling to the true vacuum, the instanton gets the value ϕ_T, and a single bubble is produced. Similar to the case in the GR theory, the instanton (or bounce) action is given by $B=S-S_F$—i.e., the difference between the action associated with the bounce solution and the false vacuum. This action determines the probability of tunneling for the process. We have defined $V_F=V(\phi_F)$ and $V_T=V(\phi_T)$ as the false and true vacuum energies, respectively. Under the approximation that the bubble wall is infinitesimally thin, we obtain the reduced action for the thin-wall bubble:

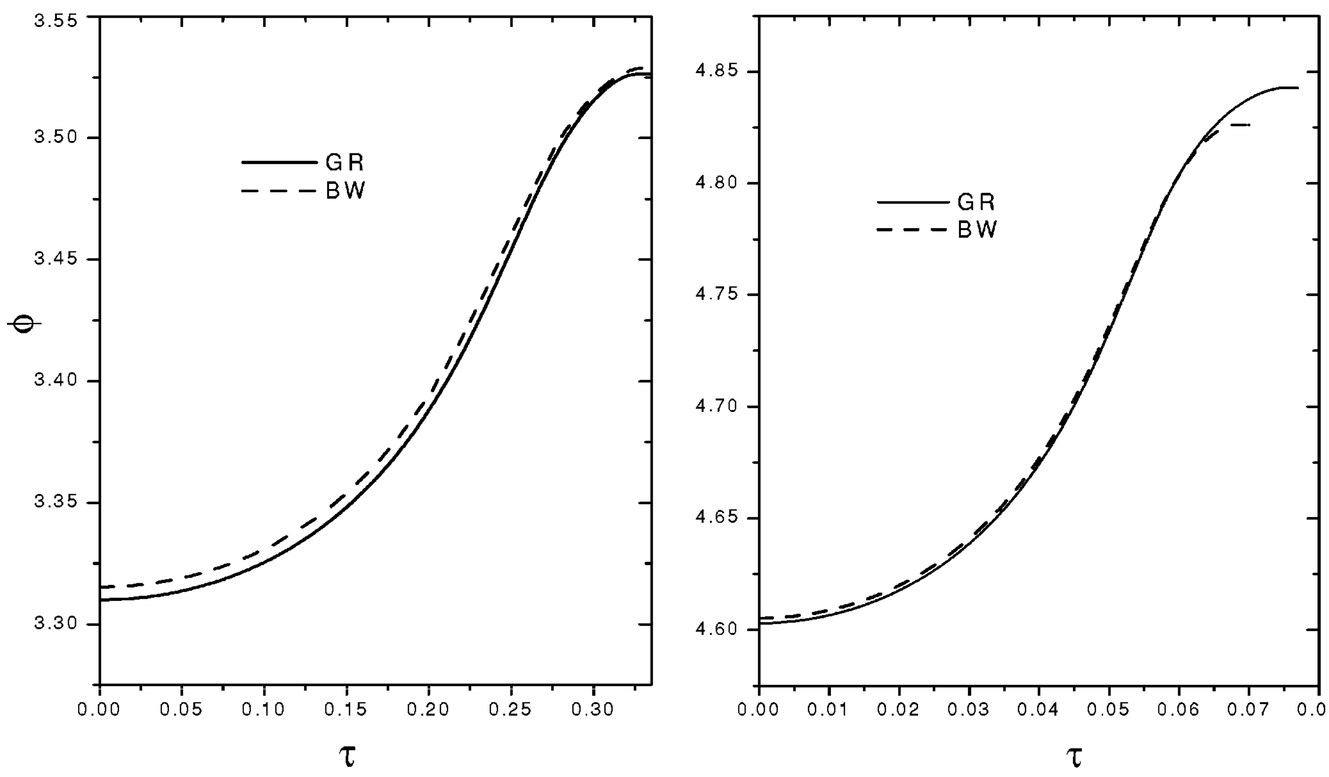

FIG. 1. The instanton $\phi(\tau)$ as a function of the Euclidean time τ is shown for Einstein GR and BW. The left panel shows the case $n=2$, and the right panel shows the case $n=4$. In both cases we have taken $\sigma=10^{-10}$.

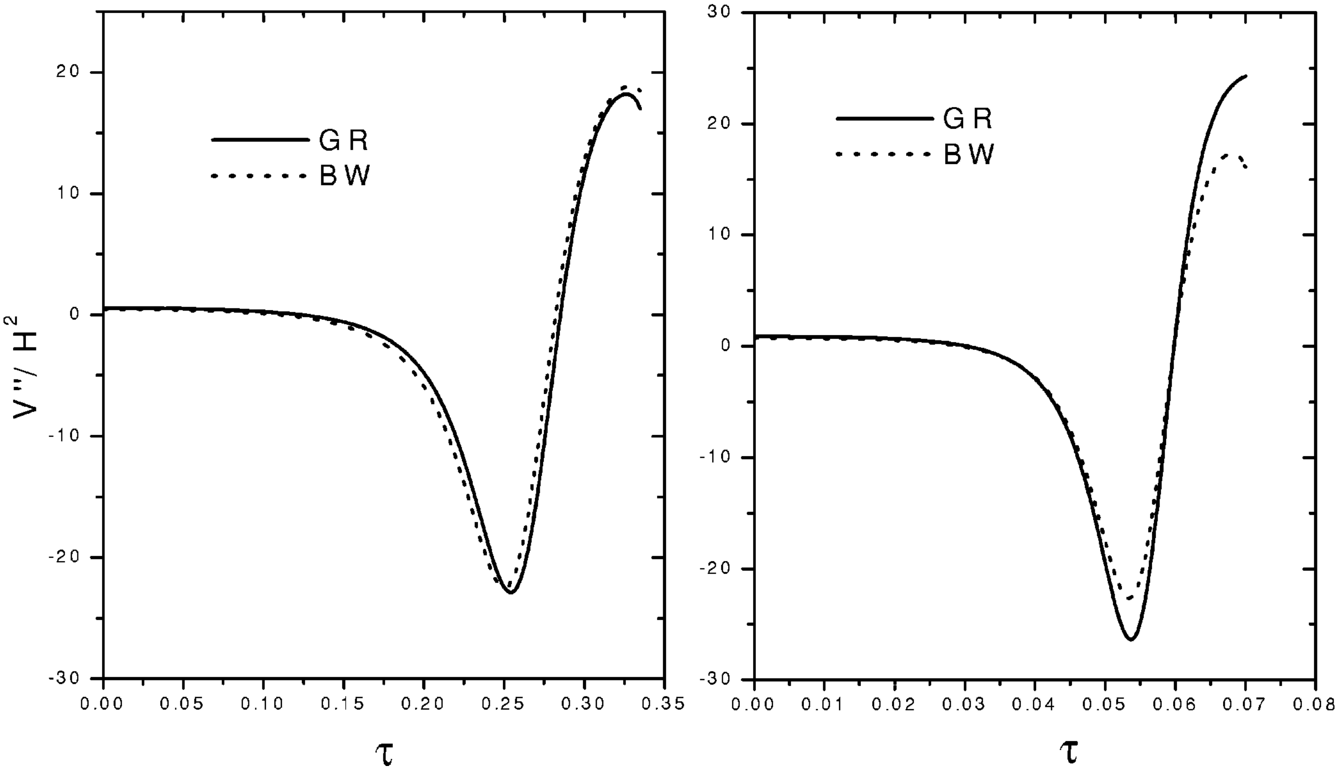

FIG. 2. This plot shows how during the tunneling process the inequality $|V''|\gg H^2$ holds. The GR lines represent the same inequality for Einstein's general theory of relativity. The left panel shows the case $n=2$ and the right panel the cases $n=4$. In both cases we have taken $\sigma=10^{-10}$.

$$S=2\pi^2R^3S_1+\frac{4\pi^2}{\kappa}\left[\frac{1}{H_T^2}([1-H_T^2R^2]^{3/2}-1)-\frac{1}{H_F^2}([1-H_F^2R^2]^{3/2}-1)\right], \tag{17}$$

where we have taken into account the contributions from the wall (first term) and the interior of the bubble (the second and third terms). Here R is the radius of the bubble, and

$$H_F^2=\frac{\kappa V_F}{3}\left[1+\frac{V_F}{2\sigma}\right], \quad H_T^2=\frac{\kappa V_T}{3}\left[1+\frac{V_T}{2\sigma}\right].$$

The surface tension of the wall becomes defined by

$$S_1=2\pi^2R^3\int d\tau\left[\phi'^{\,2}\left(1+\frac{V(\phi)}{\sigma}\right)\right] \tag{18}$$

or

$$S_1=\int_{\phi_T}^{\phi_F}d\phi\{2[V(\phi)-V_F]\}^{1/2}\left[1+\frac{V(\phi)}{\sigma}\right].$$

The radius of curvature of the bubble is one for which the bounce action (17) is an extremum. Then, the wall radius is determined by setting $dS/dR=0$, which gives

$$\frac{S_1R\kappa}{2}=(1-H_T^2R^2)^{1/2}-(1-H_F^2R^2)^{1/2}.$$

This could be solved for the radius of the bubble, and we found that

$$R=\frac{S_1\kappa}{\sqrt{\left[\left(\frac{S_1\kappa}{2}\right)^2+H_T^2+H_F^2\right]^2-4H_F^2H_T^2}} \tag{19}$$

or, equivalently,

$$R=\frac{S_1\kappa}{\sqrt{\left[\left(\frac{S_1\kappa}{2}\right)^2-H_T^2+H_F^2\right]^2+H_T^2(S_1\kappa)^2}}. \tag{20}$$

It is straightforward to check that when $\sigma\rightarrow\infty$, a correct limit to GR is obtained. We can introduce a dimensionless quantity Δs, which represents the strength of the wall tension in the thin-wall approximation [15]:

$$\Delta s=\frac{S_1R\kappa}{2}<1. \tag{21}$$

Following the values given in Table I and assuming the value $\sigma=10^{-10}$, we find that the difference in the strength of the wall tension in the thin-wall approximation becomes Δs_{BW}, which can be compared to the corresponding value in Einstein's GR theory, Δs_{GR}. For the first model—i.e., $n=2$—it becomes $\Delta s_{BW}-\Delta s_{GR}\simeq 6.08\times10^{-3}$. In the second model, when $n=4$ this difference becomes on the order of Δs_{BW}

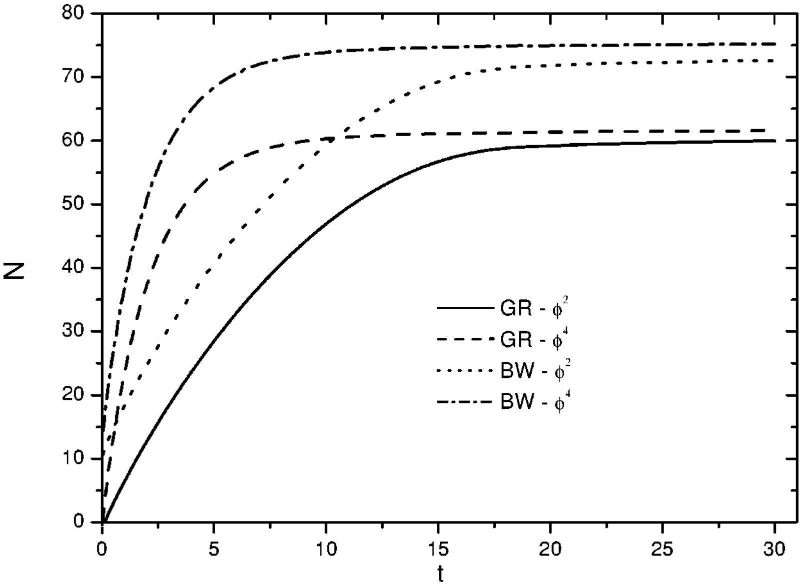

FIG. 3. This plot shows the number N of e-folds as a function of the cosmological times t. The GR line represents Einstein's GR theory. In this graph we have assumed that the constant $\sigma = 10^{-10}$.

$-\Delta s_{GR} \simeq 1.01\times 10^{-8}$. Then, we can see that this difference in the strength of the wall tension for the two theories becomes insignificant.

III. INFLATION AFTER TUNNELING

After the tunneling has occurred, we make an analytical continuation to the Lorentzian space-time, and we can see what is the time evolution of the scalar field $\phi(t)$ and the scale factor $a(t)$. The field equations of motion for the fields ϕ and a are given by

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}+V_{,\phi}(\phi)=0 \tag{22}$$

and

$$\frac{\ddot{a}}{a}=-\frac{8\pi}{3M_4^2}\left(\dot{\phi}^2-V(\phi)+\frac{1}{8\sigma}[5\dot{\phi}^2-2V(\phi)][\dot{\phi}^2+2V(\phi)]\right), \tag{23}$$

where the overdots now denote derivatives with respect to cosmological time.

In order to solve this set of equations numerically, we use the following boundary conditions: $\dot{\phi}(0)=0$, $a(0)=0$ and $\dot{a}(0)=1$. As in [3,5] and [6], the scalar field slowly rolls down to its minimum of the effective potential, and its field starts to oscillate near this minimum. During this stage the N e-fold parameter presents different values for our models under study; those results are summarized in Fig. 3. Clearly, the N e-fold parameter increases in the BW scenarios.

IV. SPECTRUM OF THE SCALAR PERTURBATION

Even though the study of scalar density perturbations in open universes is quite complicated [4], it is interesting to give an estimation of the standard quantum scalar field fluctuations inside the open bubble. The corresponding density perturbation in the brane world cosmological model becomes [11]

$$\frac{\delta\rho}{\rho}\approx Cte*\left(\frac{V}{V'}\right)^{3/2}\left(\frac{2\sigma+V}{2\sigma}\right)^{3/2}, \tag{24}$$

where $Cte=\frac{24}{5}\sqrt{8\pi/3}$. Note that the latter equation coincides with its analogous Einstein equations, when the limit $\sigma\to\infty$ is taken. Certainly, other contributions must be added in order to get an exact expression [3,4], but those contributions do not change expression (24) significantly if we use it for $N>3$.

Figure 4 shows the magnitude of the scalar perturbations $\delta\rho/\rho$ for our models as a function of the N e-folds of inflation, after the open universe was formed. Even though the shape of the graph is similar to that of Einstein's GR theory, the maximun value of $\delta\rho/\rho$ has become bigger in the BW than in Einstein's GR models. For instance, in Einstein's GR they it becomes maximum for $N\sim 0(12)$, while for the BW model its maximum is found at $N\sim 0(20)$. Also in the model with $n=4$, the values of N where $\delta\rho/\rho$ vanishes become bigger in BW than in Einstein's GR theory. On the other hand, we should mention that there is relationship between the values of N and the scale where $\delta\rho/\rho$ is measured. For $N\sim 10$, where $\delta\rho/\rho$ gets it maximum value, it is found that the scale where the scalar perturbation is measured corresponds to the 10^{24} cm. However, for $N\sim 15$ it decreases to 10^{22} cm, and for $N\gg 50$ this practically becomes to zero. Something similar happened with for the case $n=4$. There, the corresponding values of N were smaller.

Also, it is interesting to give an estimation of the tensor spectral index n_T in the brane world cosmological model. Using Ref. [11], this index for a flat universe is given by

$$n_T\simeq -\frac{1}{8\pi}\left(\frac{V'}{V}\right)^2\left[1+\frac{V}{2\sigma}\right]^{-1}. \tag{25}$$

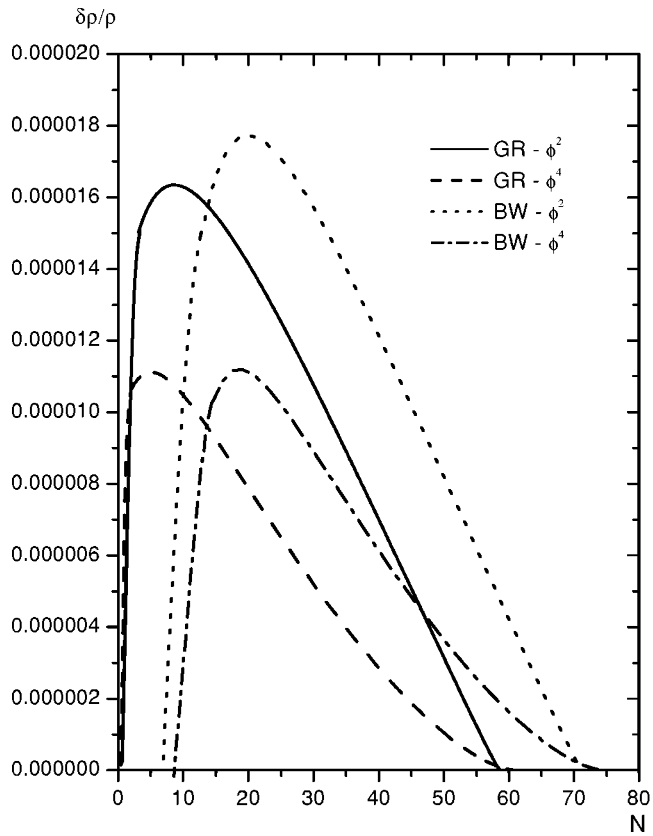

FIG. 4. Scalar density perturbations for our models as a function of *N* e-folds. These plots are compared with those obtained by using Einstein's GR theory, where $\delta\rho/\rho \approx CteH^2/|\dot{\phi}|$.

By numerically solving the field equation associated with field ϕ, we obtain for the cases $n=2$ and $n=4$ in the GR theory the following values: $n_T \simeq -0.0350$ which is evaluated in the value the N, where $\delta\rho/\rho$ presents a maximum—i.e., $N \simeq 9$ and $n_T \simeq -0.0610$ for $N \simeq 5$. In BW cosmological models for the cases $n=2$ and $n=4$, we obtain $n_T \simeq -0.0315$ for $N \simeq 20$ and $n_T \simeq -0.0611$ for $N \simeq 19$.

V. CONCLUSION

In this paper we have studied one-field open inflationary universe models in which the gravitational effects are described by BW cosmology. In this kind of theory, the Friedmann equation gets modified by an additional term $\rho^2/2\sigma$. We have solutions to an effective potentials in which the CDL instantons exist [6]. The existence of these instantons becomes guaranteed since the inequality $|V''| > H^2$ is satisfied, and thus, with the slow-roll approximation, inflationary universe models are realized for different values of the parameter n. For the two values ($n=2$ and $n=4$) that we considered, V'' remains greater than H^2 during the first e-folds of inflation.

Also, we have generalized the CDL instanton action to brane world cosmology. This action is described by expression (15).

On the other hand, it seems that, according to Eqs. (17) and (18), the result for the probability of nucleation of a bubble is the same as in Einstein gravity. However, this is superficial, since there is a modified relationship between the Hubble rate and the potential $V(\phi)$, given by the well-known modified Friedmann equation, and thus a modified expression between the wall tension S_1 and the potential $V(\phi)$ occurs. In the thin-wall limit, we have found that the strength of the wall tension—Δs_{BW}—minimum increase, when compared with their analogous results obtained in Einstein's GR theory.

We have also found that the inclusion of the additional term (ρ^2) in the Friedmann's equation improves some of the characteristic parameters of inflation. For instance, this is accentuated in the number N the e-folds (see Fig. 3).

Finally, in the $\delta\rho/\rho$ graphs the maximum presents a displacement when compared with that obtained in Einstein's GR theory; this would change the value of the fundamental parameter λ_n that appears in the scalar potentials.

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